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Vittorio Giovannetti

Quantum Energy Lines
and the optimal output
ergotropy problem

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arXiv:2104.11255v2 [quant-ph] 22 Jul 2021

Quantum Energy Lines and the optimal output ergotropy problem

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We study the transferring of useful energy (work) along a transmission line that allows for partial preservation of quantum coherence. As a figure of merit we adopt the maximum values that ergotropy, total ergotropy, and non-equilibrium free-energy attain at the output of the line for an assigned input energy threshold. For Phase-Invariant Bosonic Gaussian Channels (BGCs) models, we show that coherent inputs are optimal. For (one-mode) not Phase-Invariant BGCs we solve the optimization problem under the extra restriction of Gaussian input signals.

NEST

SCUOLA NORMALE SUPERIORE

Quantum Technologies

Equilibration/Thermalization

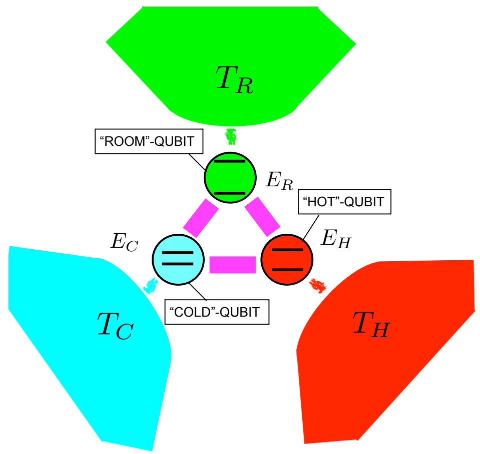
Processes

Quantum Thermodynamics

Goold et al. JMP 49 (2016)

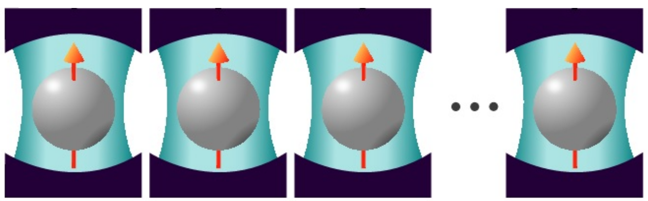
Quantum Engines

(Energy production)

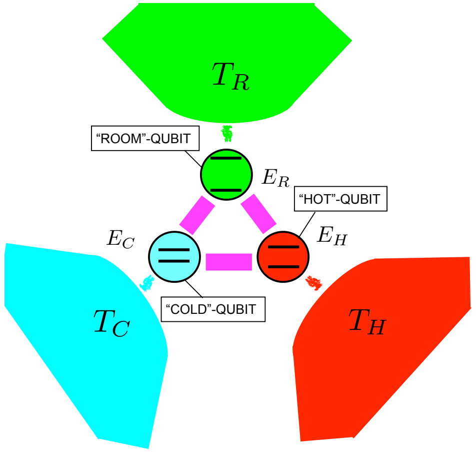


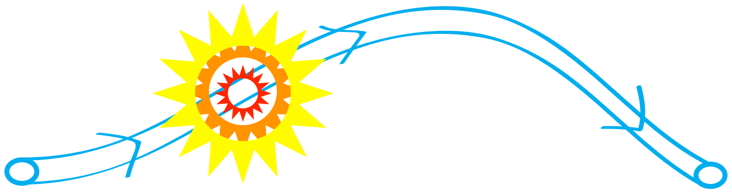
Quantum Batteries

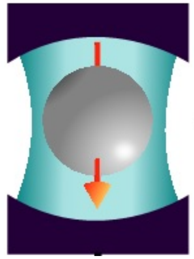
(Energy storage)



Campaioli, et al. arXiv:1805.05507 **[quant-ph]**







Quantum Energy Lines

(Energy transmission)

Can we use "quantum signals" to transfer energy?

What are the suitable (realistic) models?

Are they efficient?

What are the proper figure of merit to address the problem?

Our Results (in brief)

- A CV variable model for QEL (e.m. transmission lines)
- Show that COHERENT states of the radiation are optimal inputs: they allow one to transfer the maximum amount of "useful energy"

Our Results (in brief)

1

- A CV variable model for QEL (e.m. transmission lines)

3

- Show that COHERENT states of the radiation are optimal inputs: they allow one to transfer the maximum amount of "useful" energy

2

1

QUANTUM ENERGY LINES

QUANTUM ENERGY LINES

“Any physical media that allows one to transfer energy from one location A to a second location B while maintaining a *certain* degree of quantum coherence in the transmitted signals.”

Optical fibers

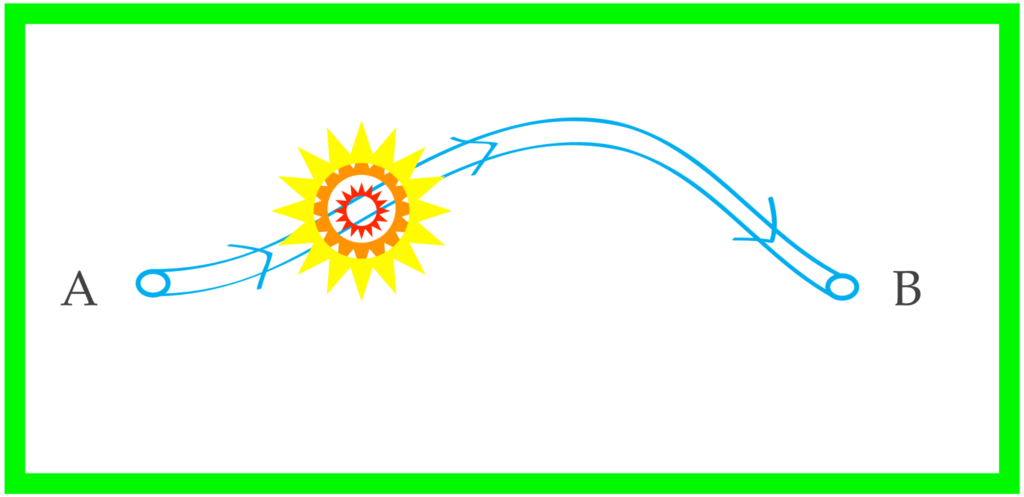
Wave guides

Free-space e.m. communication

Nanowires

Metallic leads

...

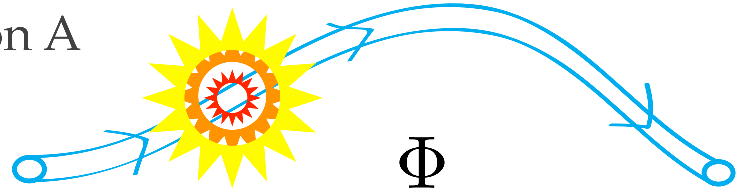


Quantum Channel formalism
(CPT formalism)

<== **QUANTUM COMMUNICATION**

Location A | Location B

$\hat{\rho}$



Φ

$\hat{\rho}' = \Phi[\hat{\rho}]$

Input signal | Output signal

Φ = transformation which maps the (quantum) state of the input signals into the associated (quantum) states at the output of the communication line

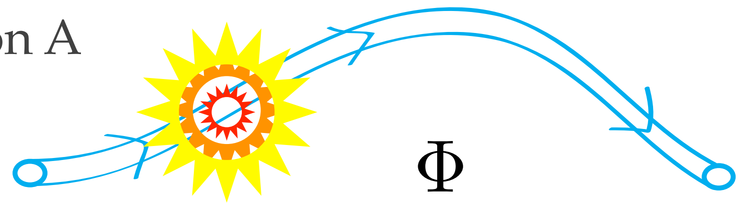
TRACE PRESERVING
COMPLETELY POSITIVE
LINEAR SUPEROPERATORS

Generalization of the notion of SCATTERING matrices, to the case where signals undergo to NOISE (fluctuations, thermalization effects, spurious collisions, losses, etc.)

Physical Model of the QEL

Location A

$\hat{\rho}$



Φ

Location B

$\hat{\rho}' = \Phi[\hat{\rho}]$

Input signal

$(\hat{H} \geq 0)$

Output signal

$$E_{in} = \mathrm{Tr}[\hat{\rho}\hat{H}] \longrightarrow E_{out} = \mathrm{Tr}[\hat{\rho}'\hat{H}] = \mathrm{Tr}[\Phi[\hat{\rho}]\hat{H}]$$

Which input signals with assigned input energy E produce output signals that ensure the highest value of "useful energy"?

2

“USEFUL” ENERGY

“USEFUL” ENERGY = <u>EXTRACTABLE WORK</u>

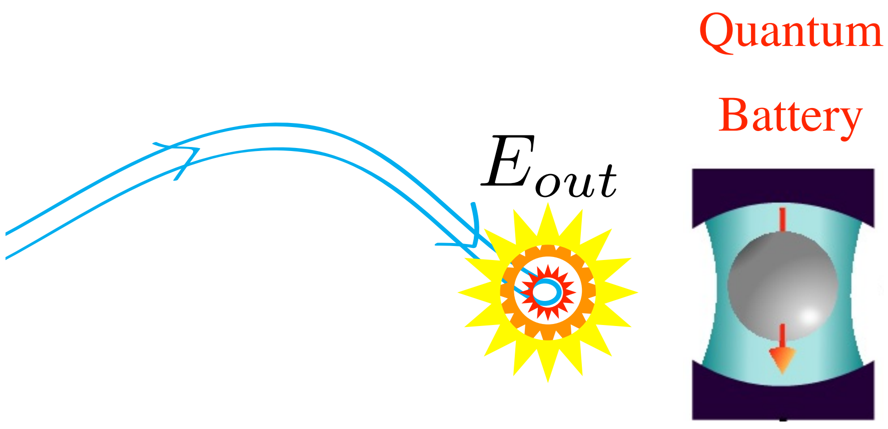
$$\Delta E = Q - W$$

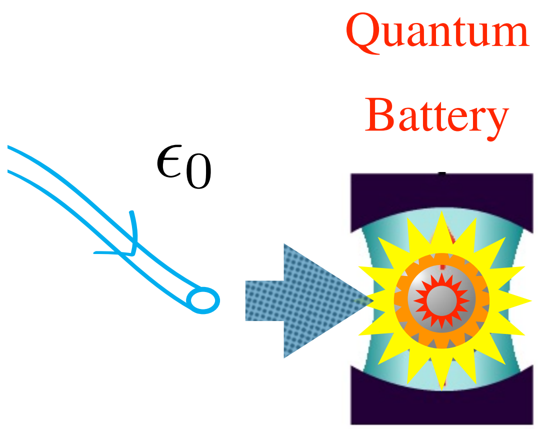
Variation of the Internal energy of A system

Heat added to the system

Work done by the system

“USEFUL” ENERGY = EXTRACTABLE WORK

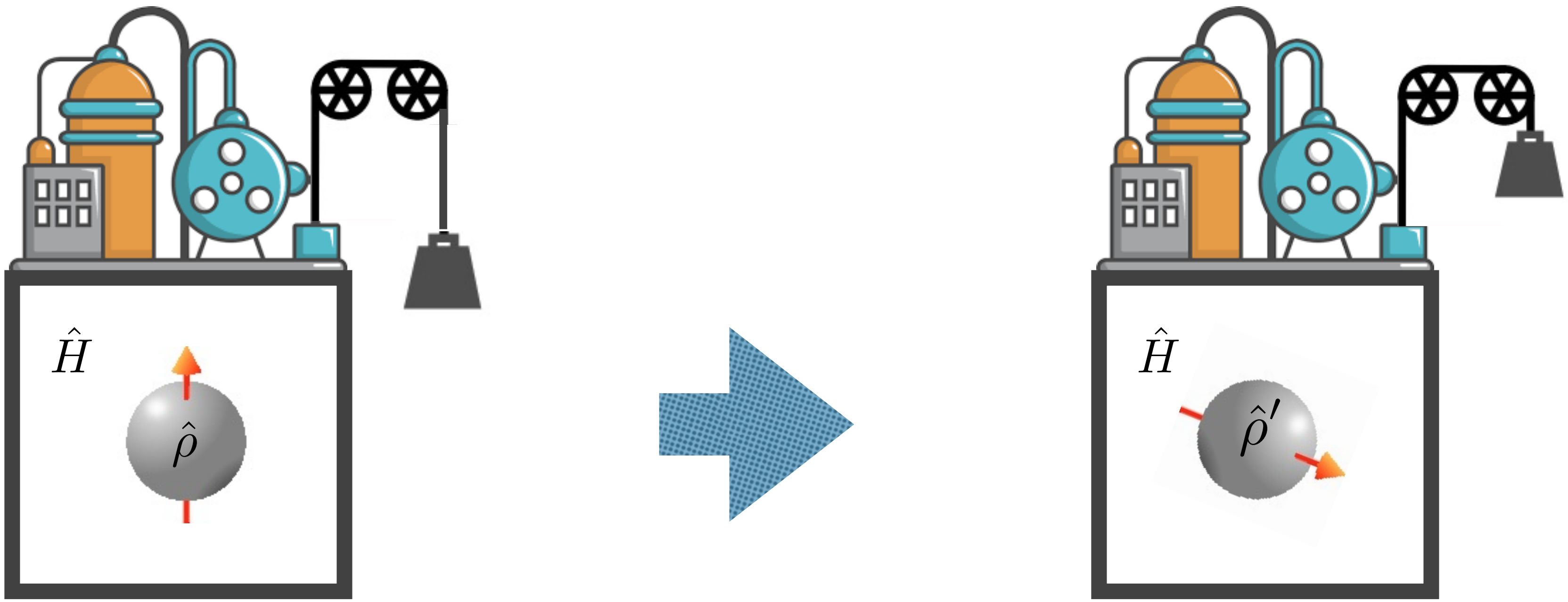




$$\Delta E = E_{out} - \epsilon_0 \stackrel{?}{=} Q - W$$

(ground state energy of the energy carrier)

“USEFUL” ENERGY = EXTRACTABLE WORK in a QUANTUM SETTING

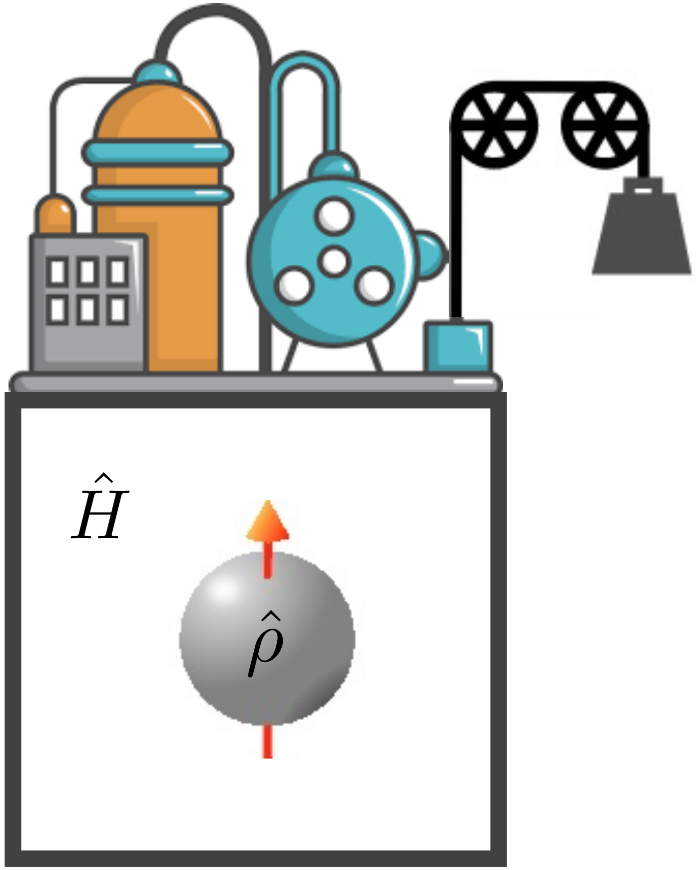


$E(\hat{\rho}) = \mathrm{Tr}[\hat{\rho}\hat{H}]$ $\qquad$ $E(\hat{\rho}') = \mathrm{Tr}[\hat{\rho}'\hat{H}]$

$$\Delta E = E(\hat{\rho}') - E(\hat{\rho}) \stackrel{?}{=} Q - W$$

The answer depends on the type of allowed transformation and on the resources we invest in the process

WE FOCUS ON THREE DIFFERENT SCENARIOS



$$E(\hat{\rho}) = \mathrm{Tr}[\hat{\rho}\hat{H}]$$

1) If you allow only MODULATIONS of the system Hamiltonian (no thermal contact with external baths),

EW= ERGOTROPY

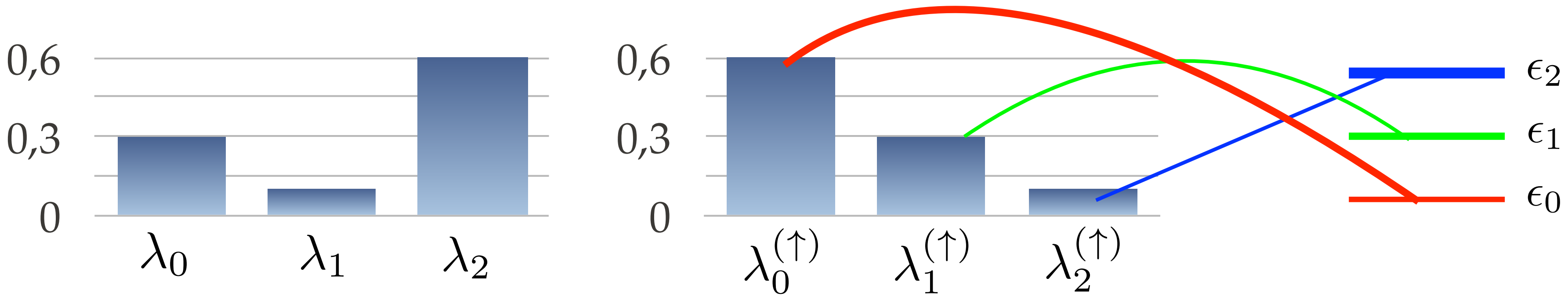
$$\mathcal{E}(\hat{\rho}) := \mathrm{Tr}\left[\hat{\rho}\hat{H}\right] - \min_{V} \mathrm{Tr}\left[V\hat{\rho}V^{\dagger}\hat{H}\right]$$

Lenard JSP 19 (1978)
Petz & Woronowicz CMP (1978)
Alicki JPA (1979)
Allahverdjan et al. EPL 67 (2004)

ERGOTROPY

NON LINEAR functional of $\hat{\rho} = \sum_{j=0} \lambda_j |j\rangle\langle j|$ and $\hat{H} = \sum_{j=0} \epsilon_j |\epsilon_j\rangle\langle \epsilon_j|$

$$\mathcal{E}(\hat{\rho}) = E(\hat{\rho}) - \sum_{j=0} \lambda_j^{(\uparrow)} \epsilon_j$$



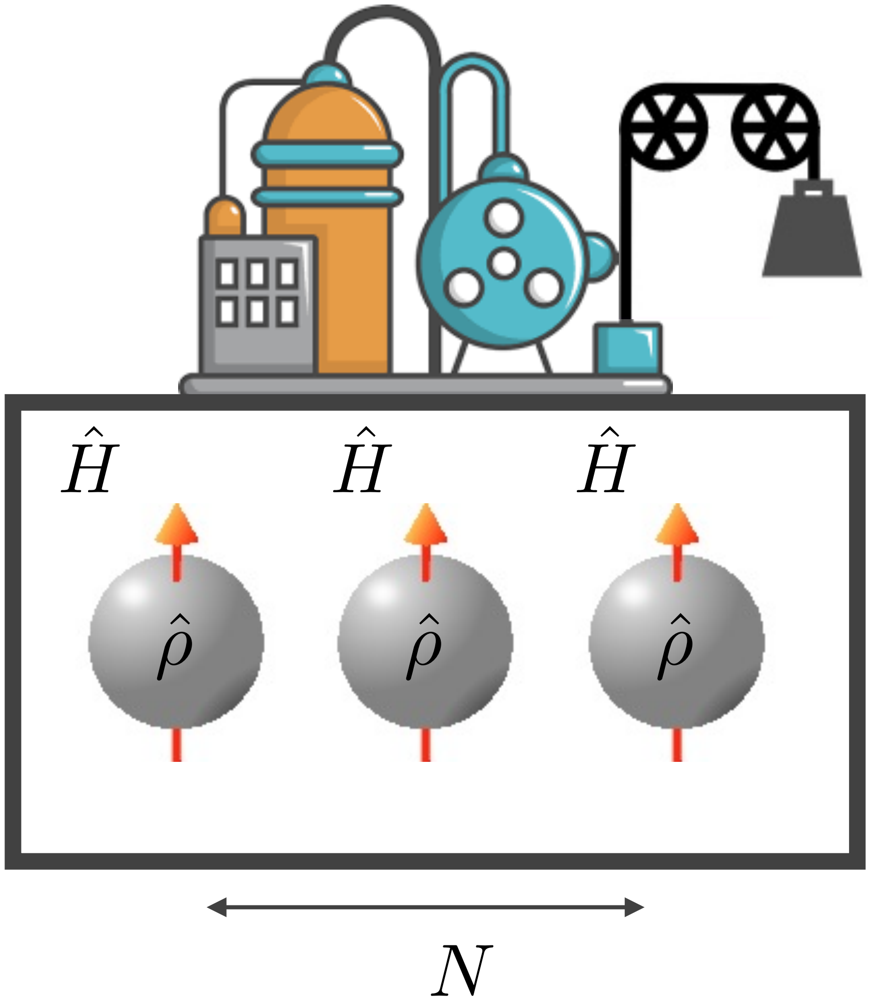
$$0 \leq \mathcal{E}(\hat{\rho}) \leq E(\hat{\rho}) = \mathrm{Tr}[\hat{\rho}\hat{H}]$$

$$\mathcal{E}(\hat{\rho}) = 0$$

for PASSIVE states (i.e. diagonal in the energy eigen-basis with decreasing spectrum)

$$\mathcal{E}(\hat{\rho}) = E(\hat{\rho})$$

if and only if the state is PURE

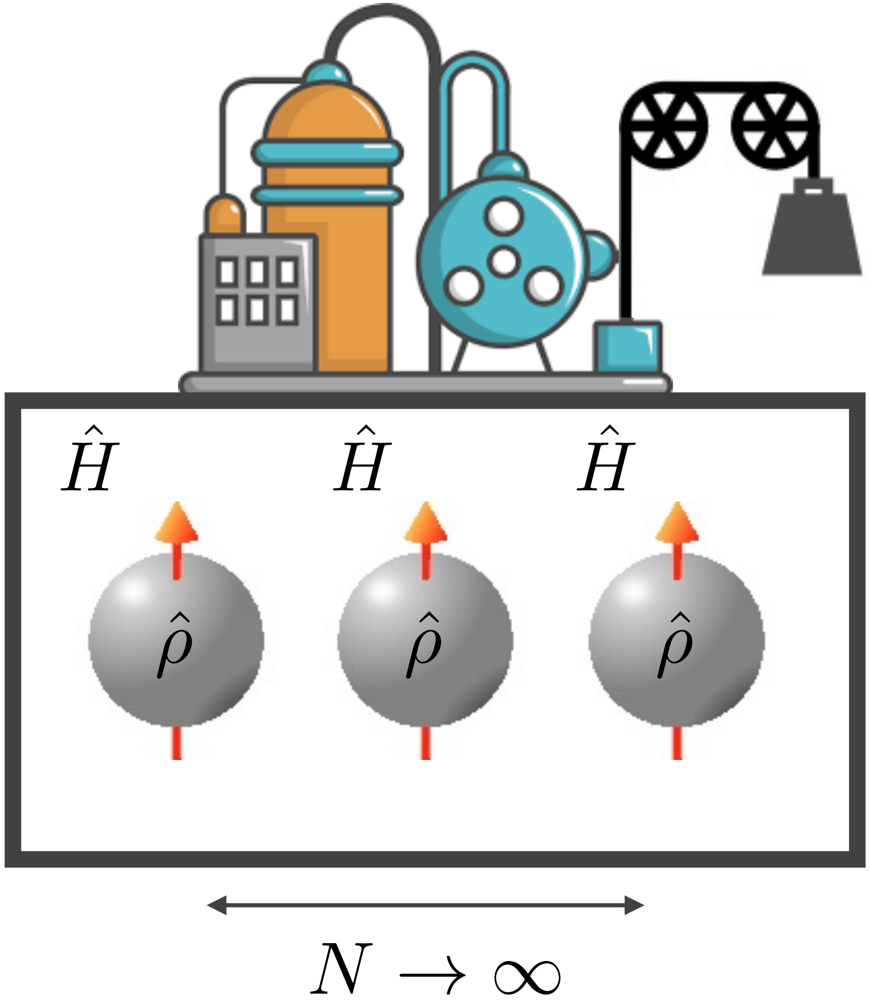


2) If you allow only MODULATIONS of the system Hamiltonian of N copies of the system (no thermal contact with external baths),

EW per copies = N-ERGOTROPY

$$\boxed{\mathcal{E}^{(N)}(\hat{\rho}) = \frac{\mathcal{E}(\hat{\rho}^{\otimes N})}{N}} \geq \mathcal{E}(\hat{\rho})$$

Lenard JSP 19 (1978)
Petz & Woronowicz CMP (1978)
Alicki JPA (1979)
Allahverdjan et al. EPL 67 (2004)



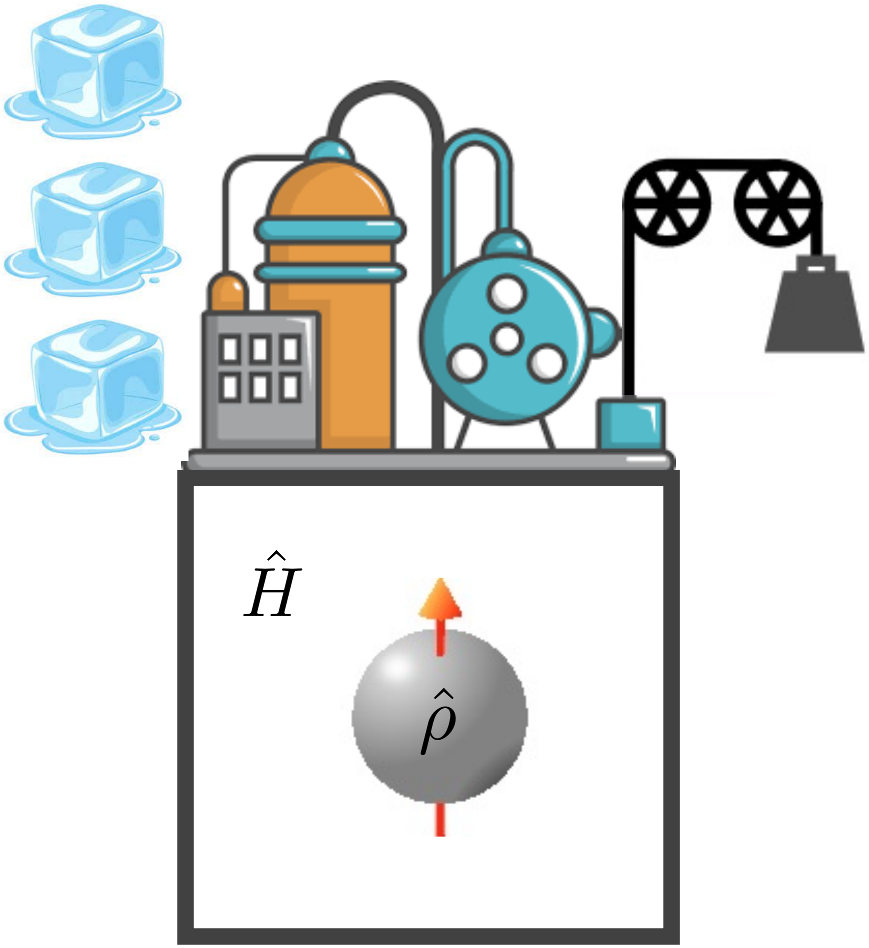
TOTAL ERGOTROPY

$$\mathcal{E}_{tot}(\hat{\rho}) = \lim_{N\to\infty} \mathcal{E}^{(N)}(\hat{\rho}) = E(\hat{\rho}) - E(\hat{\omega}_{\bar{\beta}})$$

Gibbs Thermal
state of the
system
with THE SAME
entropy of $\hat{\rho}$

$$\hat{\omega}_{\beta} = \frac{e^{-\beta\hat{H}}}{Z(\beta)}$$

$$S(\hat{\omega}_{\bar{\beta}}) = S(\hat{\rho})$$



3) If you allow the agent to put the system in thermal contact with an external source at temperature $T = 1/\beta$

EW= NON EQUILIBRIUM FREE ENERGY

$$\mathcal{F}_\beta(\hat{\rho}) = E(\hat{\rho}) - \frac{1}{\beta}S(\hat{\rho})$$

Brandao et al, PNAS 112 (2015)

N.B. for *T=0* $\mathcal{F}_\beta(\hat{\rho}) \to E(\hat{\rho})$

THE PROBLEM

FIND which are the input states with fixed input energy that yield the maximum value of the EW at the output of the transmission line

$$EW_E(\Phi) = \max_{\hat{\rho} \in \mathcal{H}_E} EW(\Phi(\hat{\rho})) = ?$$

$$\mathcal{H}_E = \{\hat{\rho} : E(\hat{\rho}) \leq E\}$$

EW= ERGOTROPY

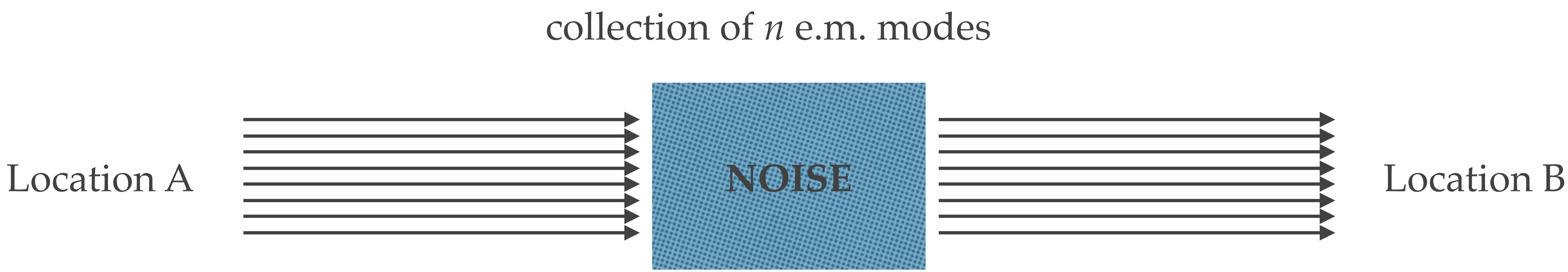
EW= TOTAL ERGOTROPY

EW= NON EQUILIBRIUM FREE ENERGY

3

RESULTS

"PHASE-INSENSITIVE GAUSSIAN BOSONIC CHANNELS"



$\hat{a}_i, \hat{a}_i^\dagger$ = annihilation and creation operators of the i-th mode $(i = 1, \cdots, n)$

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$$

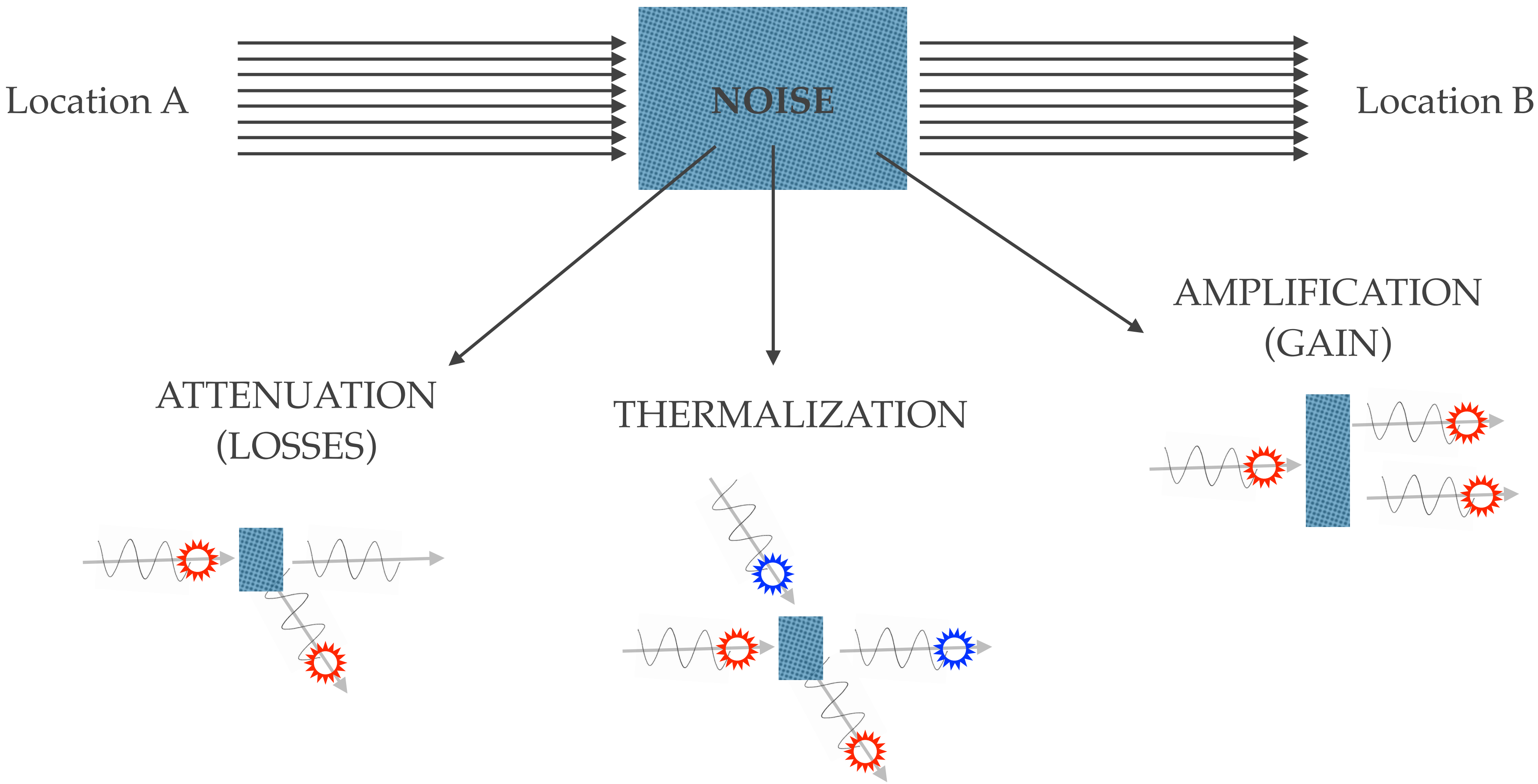
$$[\hat{a}_i, \hat{a}_j] = 0$$

$$\hat{H} = \sum_{i=1}^{n} \hbar\omega_i \hat{a}_i^\dagger \hat{a}_i$$

ω_i = frequency of the i-th mode

$$(\omega_i \simeq \omega_0)$$

collection of n e.m. modes



"PHASE-INSENSITIVE GAUSSIAN BOSONIC CHANNELS"

Holevo and Giovannetti RMP (2012)

Given Φ a PI-BGC , for any E the maximum output Extractable Work

$$EW_E(\Phi) = \max_{\hat{\rho} \in \mathcal{H}_E} EW(\Phi(\hat{\rho}))$$

is achieved by a (multimode) coherent state

(multimode) COHERENT STATES

$$\hat{\rho}_{cohe} = |\Psi\rangle\langle\Psi|$$

$$|\Psi\rangle = \hat{D}(\vec{m})|\emptyset\rangle$$

DISPLACED VACUUM STATES

Sketch of the proof:

0) Notice that all our 3 examples of EW can be expressed as

$$EW(\Phi(\hat{\rho})) = E(\Phi(\hat{\rho})) - (\text{something} \geq 0)$$

with the something terms being all Shur concave functionals of the input state.

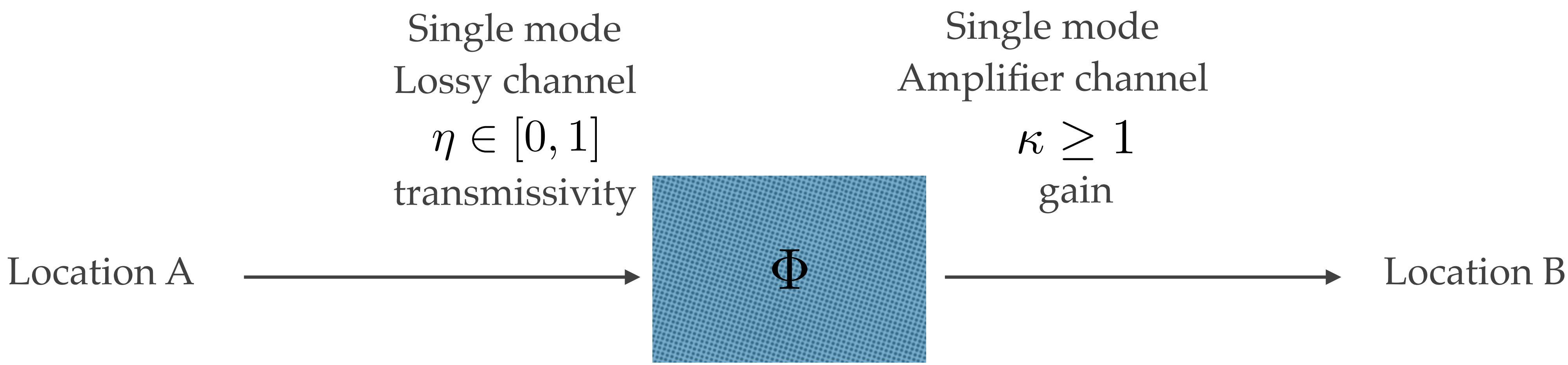
1) show that for any input state $\hat{\rho}$ there exists a properly chosen coherent input $\hat{\rho}_{cohe}$ with the same initial energy which at the output of the channel provides a signal with mean energy that is larger than or equal to the output mean energy of $\hat{\rho}$

$$E(\hat{\rho}) = E(\hat{\rho}_{cohe}) \qquad E(\Phi(\hat{\rho})) \leq E(\Phi(\hat{\rho}_{cohe}))$$

2) invoke the fact that at the output of a PI-GBC coherent inputs MAJORIZE (*are more ordered than*) all other signals

Mari, Giovannetti, Holevo Nat. Comm. (2014)

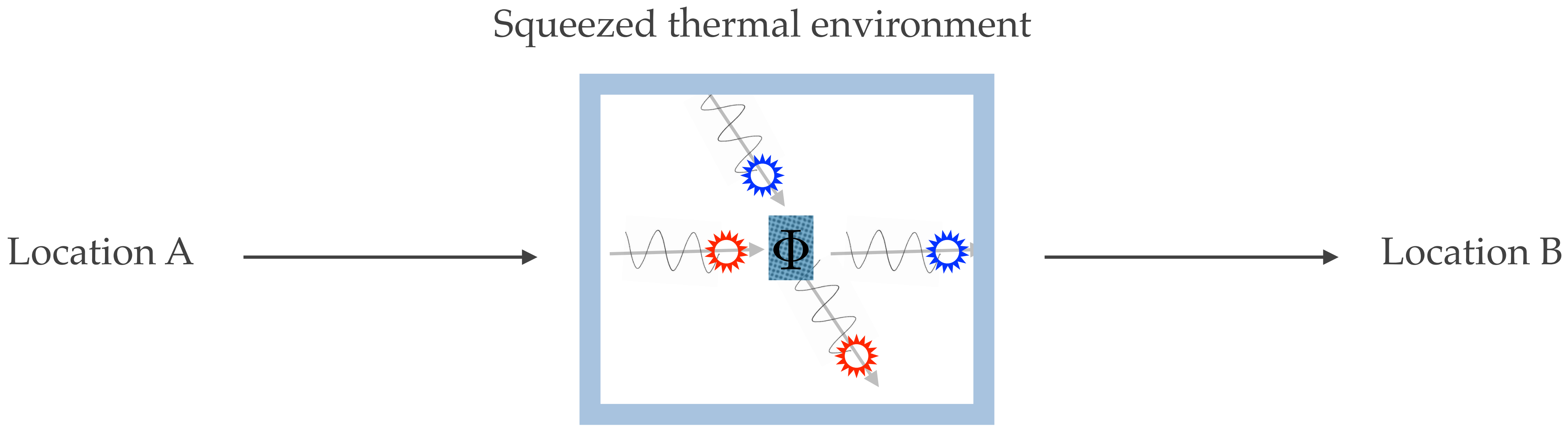
Examples



$$EW_E(\Phi) = \eta E$$

$$EW_E(\Phi) = \kappa E$$

GAUSSIAN BOSONIC CHANNELS
Which are NOT PHASE-INSENSITIVE



Optimal inputs are not known in this case

Coherent input states ARE certainly NOT optimal

Properly squeed inputs for instance perform better than coherent states

CONCLUSIONS

- QEL models have being introduced
- Optimization problem has been presented based on the notion of EW
- For CV systems (realistic noise model) optimal solution have been presented

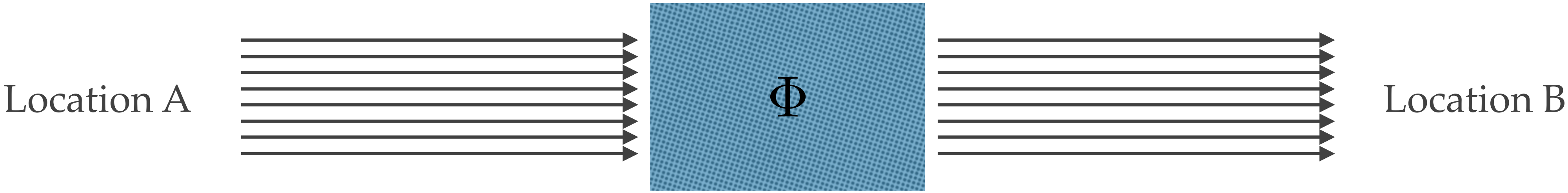
FUTURE DIRECTIONS

- Extended the analysis to other QEL models
- Introduce different figure of merit (power, instead of energy, work distribution)

4

Extras

collection of n e.m. modes



PHASE-INSENSITIVE GAUSSIAN BOSONIC CHANNELS

(loss, thermalization,amplification noise, etc)

$$\hat{q}_i = \frac{\hat{a}_i + \hat{a}_i^\dagger}{\sqrt{2}}$$

$$\hat{p}_i = i\frac{\hat{a}_i^\dagger - \hat{a}_i}{\sqrt{2}}$$

$$\hat{r} = (\hat{q}_1, \cdots, \hat{q}_n; \hat{p}_1, \cdots, \hat{p}_n)^T$$

vector of the canonical coordinates

$$\hat{D}(\vec{x}) = \exp[\hat{r} \cdot \vec{x}]$$

displacement operator

$\vec{x} \in R^{2n}$

$$\chi(\hat{\rho}; \vec{x}) = \mathrm{Tr}[\hat{\rho}\hat{D}(\vec{x})]$$

CHARACTERISTIC FUNCTION

(Fourier transform of the Wigner distribution of the state)

$$\chi(\hat{\rho};\vec{x})$$

GAUSSIAN BOSONIC CHANNELS

$$\chi(\Phi(\hat{\rho});\vec{x}) = \chi(\hat{\rho}; X^T\vec{x})\exp[-\frac{1}{4}\vec{x}^T Y \vec{x} + i\vec{v}^T\vec{x}]$$

$$\vec{v} \in R^{2n}$$

$$X, Y \in R^{2n\times 2n} \qquad Y \geq i(\gamma - X\gamma X^\dagger)$$

(symplectic matrix)

$$\Phi(e^{-i\hat{H}t}\hat{\rho}e^{i\hat{H}t}) = e^{-i\hat{H}t}\Phi(\hat{\rho})e^{i\hat{H}t}$$

PHASE-INSENSITIVE
Condition

Bosonic Gaussian Channels (BGC)

"Completely Positive Trace Preserving (CPTP) Super-operators Φ which map Gaussian states into Gaussian states"

$$\hat{\rho} \longrightarrow \Phi(\hat{\rho})$$

Gaussian states = Gibbs states of arbitrary Hamiltonians which are quadratic in the field operators

$$[\hat{q}, \hat{p}] = i \qquad \hat{r} := (\hat{q}, \hat{p})^T$$

Covariance matrix

$$\phi_G(\hat{\rho}; z) = \exp[-\tfrac{1}{4} z^T \sigma z + i m^T \cdot z]$$

$$\sigma \geq \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

symplectic form

characteristic function of the state

$$\phi(\hat{\rho}; z) := \mathrm{Tr}\left[\hat{\rho}\, e^{i\hat{r}^T \cdot z}\right]$$

Weil (displacement) operator

$z \in \mathbb{R}^2$

Holevo, Werner PRA 2001